GROUP THEORY: RE-EXAM 2016

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Problem 1 (20%). For p prime and $n < p$, prove that the only way in which the group $\mathbb{Z}/p\mathbb{Z}$ can act on the set of n objects is the *trivial* action: $(g, s) \mapsto s$ for all $g \in \mathbb{Z}/p\mathbb{Z}$ and any object s.

Problem 2 (20%). To prevent acts of vandalism in a museum, a photo-camera was set to watch the ancient cultural artefacts, and so it did: $\boxed{1\,2\,3\,4}$. Three hooligans called X, Y, and Z fancied that permuting old artefacts would be fun (in fact, it is not).

X:		Y:		Z:	
	$\boxed{4\,2\,1\,3}$		$\boxed{2\,3\,1\,4}$		$\boxed{4\,3\,2\,1}$
	$\boxed{4\,1\,2\,3}$		$\boxed{3\,4\,1\,2}$		$\boxed{3\,4\,1\,2}$

Here are the photo-camera shots of two permutations each of them did. Whose two permutations were *conjugate*?

Problem 3 (20%). Let $n \geqslant 4$. Prove that in the group A_n of parity-even permutations of $\{1, \ldots, n\}$, all products of two disjoint 2-cycles are pairwise conjugate.

Problem 4 (20%). Let G be a finite group and $H \preccurlyeq G$ be a subgroup of index n, so that $\sharp G = n \cdot \sharp H$. Recall that $\sharp S_n = n!$ and now prove that $g^{n!} \in H$ for every $g \in G$.

Problem 5 ($12\% + 8\%$). Consider the subgroup

$$H := \mathbb{Z} \cdot (2, 3, 3) + \mathbb{Z} \cdot (5, 5, 5) \subsetneq \mathbb{Z}^3.$$

Find (i) the order of the element $(1, 0, 0) + H$ and (ii) of the element $(0, 0, 1) + H$ in the factorgroup $\mathbb{Z}^3/H$.

Date: April 11, 2016. GOOD LUCK!